Graphical Introduction to The Derivative Function

Activity 3 is about $f(x)$ and $f'(x)$. Let $y = f(x)$ be a given function. Recall:

$y = f(x) =$ **'the height of the function at x'.**

$y' = f'(x) =$ **'the slope of the tangent line to the function at x'.**

Example:

Below (on the left) is a graph of the function $y = f(x) = x^3 + x^2 - 4x + 1$. I have plotted several heights. For example:

$y = f(0) = (0)^3 + (0)^2 - 4(0) + 1 = 1 =$ 'height at $x = 0$'

$y = f(1) = (1)^3 + (1)^2 - 4(1) + 1 = -1 =$ 'height at $x = 1$'

$y = f(2) = (2)^3 + (2)^2 - 4(2) + 1 = 5 =$ ' height at $x = 2$'

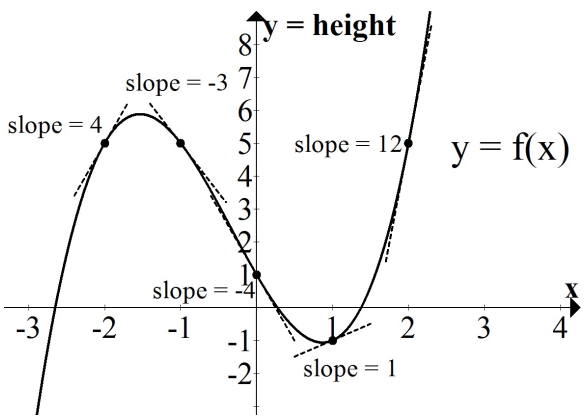
At several values, you will see the tangent slope drawn. We have learned that $y' = f'(x) = 3x^2 + 2x - 4$. For example:

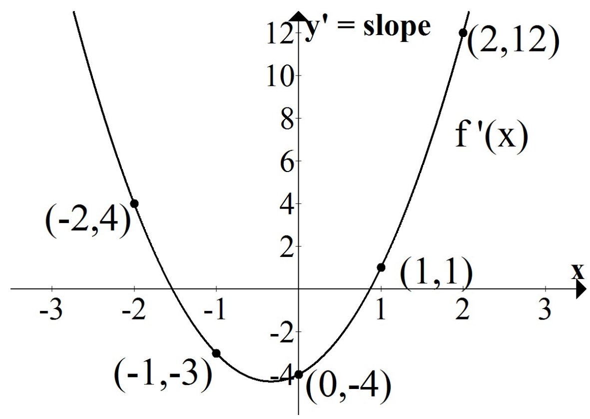
$y' = f'(0) = 3(0)^2 + 2(0) - 4 = -4 =$ 'slope at $x = 0$'

$y' = f'(1) = 3(1)^2 + 2(1) - 4 = 1 =$ 'slope at $x = 1$'

$y' = f'(2) = 3(2)^2 + 2(2) - 4 = 12 =$ 'slope at $x = 2$'

On the right, we have plotted these slopes in a new graph. We call this the derivative graph.





The goal of Activity 3 is to start to see the connections between these two graphs. Notice that the graphs do NOT look the same and they are giving very different information about the function. But there are a few key connections between the graphs. Here are the connections I want you to think about:

ORIGINAL ($f(x)$)	DERIVATIVE ($f'(x)$)
$f(x) =$ height of original at x	$f'(x) =$ slope of original at x
increasing (uphill left-to-right)	positive (above x-axis)
decreasing (downhill left-to-right)	negative (below x-axis)
horizontal tangent	zero (crosses x-axis)

The facts above are fundamental to calculus and we will use them over and over and over again in applications. Eventually you will be able to use these even if you don't have a graph in front of you. These connections should be something you natural use without thinking about by the end of the term. At the end of this activity, you should have some strategies for the following two tasks:

1. Given the 'original' height graph, draw a rough sketch of the 'derived' slope graph.

2. Given the 'derived' slope graph, draw a very rough sketch of the general shape of the 'original' height graph.

Keep this sheet as a reference, but eventually you need to know these connections well without having to look at this sheet.